Hydrologic Process Parameterization of Electrical Resistivity Imaging of Solute Plumes Using POD McMC

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ABSTRACT
Markov chain Monte Carlo (MCMC) techniques are widely used in geophysics due to their ability to recover multiple probable outcomes that enable uncertainty assessment. Standard MCMC methods, however, become computationally intractable in high dimensionality spaces. We present a MCMC approach based on a sparse proper orthogonal decomposition (POD) model parameterization that implicitly incorporates the physics of the underlying process.

INTRODUCTION
Geophysical estimation of mass and solute plume morphologies is becoming increasingly popular for numerous hydrological investigations including optimal design of contaminant remediation schemes and estimation of transport parameters.

Geophysical estimation involves inherent uncertain inputs due to limited noisy measurements with incomplete understanding of subsurface processes.

Markov chain Monte Carlo (MCMC) methods enable uncertainty assessment of geophysical estimation in light of the inherent uncertain inputs.

The problem
Standard MCMC become computationally intractable in high dimensional problems, such as in spatiotemporally altered poro-hydrogeological process parameterized by 2D and 3D boreholes.

This research presents an MCMC version of the Proper Orthogonal Decomposition (POD)-based inversion algorithm (Oware et al., 2013; Oware and Mayeux, 2014).

The POD-based inversion parameterizes the problem in the reduced dimensionality space using small number of optimal basis vectors tuned to the target process as projection vectors.

Hence, a small number of inversion parameters are estimated, making MCMC in high dimensionality spaces feasible while accounting for the physics of the underlying process.

METHODS

DETAILS OF THE POD McMC ALGORITHM
- Fig. 3 presents the workflow for the proposed POD McMC algorithm.
- The POD plumes (Fig. 4) are constructed from POD decompositions of training images (TIs) obtained from a single solute source transport process in a random field.
- A projection of the TIs into the reduced optimal basis space produces \( \epsilon_{\text{prior}} \), from which prior standard deviations (or variances) for each coefficient are estimated. For instance, 400 TIs will result in 400 realizations of each coefficient, producing prior distributions for each coefficient (Fig. 5).
- The starting or reference coefficient \( \epsilon_{\text{prior}} \) structure is obtained from a projection of a homogeneous background model into the basis space.
- The Gaussian nature of the histograms in Fig. 5 supports the assumption of a Gaussian prior model, N(0,1), for the coefficient re-simulation.
- A sampling window is employed to re-simulate a subset of the total coefficients at each iteration. The sampling window grows at a specified rate to incorporate more information as iteration progresses starting from the coefficients of the low-order bases to those of the least informative bases.
- Posterior samples are obtained only after the window grows fully and all the coefficients are re-sampled jointly.

REFERENCES


RESULTS

INVERSION RESULTS FOR THE UNIMODAL TARGET PLUME WITH ACCURATE PRIOR ASSUMPTIONS
- The algorithm was run for 50,000 iterations and Acos to basis in rapidly (Fig. 6). The starting misfit model was around 90% which reduced to a stationary level of about 3.5% compared to the expected error level of 5%.
- The posterior samples were considered from 50,000 to 50,000 when the coefficient sampling window stopped growing.
- Qualitative comparison of the posterior mean and the realizations suggest the target plume was accurately estimated with minimal plume smearing (Fig. 7).
- Qualitative comparison of the prior assumption in the unimodal test case is captured in the estimated seemingly low standard deviations.

CONCLUSIONS
- We have demonstrated that McMC can be parameterized in the reduced dimensionality space while accounting for the physics of the underlying process.
- We were able to accurately capture plume morphology even if the conceptual model is not completely accurate or fully developed. Nevertheless, accurate prior information reduces uncertainty in the estimation.
- The algorithm was found to be computationally efficient, operating at 90% truncation in the dimensionality of the problem, i.e., 300 coefficients were estimated to reconstruct 3000 full dimensionality space.

REFERENCES